647510 **BSCbooCC5030** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
March/April-2022(Old Course)
BOOLEAN ALGEBRA AND COMPLEX ANALYSIS-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer any one. (07)

(1) R is an equivalence relation on set X then prove that

(i) $\bigcup_{x \in X}[x] = X$

(ii) $xRy \Leftrightarrow [x] = [y]$

(2) Let (L, $*, \oplus$) be a Lattice, a, b, c $\in$ L then prove that

$a * (b \oplus c) = (a * b) \oplus (a * c) \Leftrightarrow a \oplus (b * c) = (a \oplus b) * (a \oplus c)$

Q.1(B) Answer any one. (04)

(1) A = { a, b, c} and $\subseteq$ is a relation of subset or equal set then show that (P(A), $\subseteq$) is poset.

(2) If (L, $\leq$) is a Lattice and a, b, c $\in$ L, then prove that $b \leq c \Rightarrow a * b \leq a * c$ and $a \oplus b \leq a \oplus c$

Q.1(C) Answer any one (03)

(1) Define Cover. Find all covers in (S_{30}, D).

(2) In usual notations prove that $a * (b * c) = (a * b) * c$

Q.2(A) Answer any one. (07)

(1) If (B, $*, \oplus, ', 0, 1$) is Boolean algebra and a, b $\in$ B then prove that

$a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \leq a' \Leftrightarrow a' \oplus b = 1$.

(2) Obtain sum of product canonical form of $\alpha(x_1, x_2, x_3) = x_2 \oplus x_3$

Q.2(B) Answer any one. (04)

(1) Obtain product of sum canonical form of $\alpha(x_1, x_2, x_3) = x_1 * x_3$

(2) If (B, $*, \oplus, ', 0, 1$) is Boolean algebra and $x_1, x_2 \in$ B then prove that

$A(x_1 * x_2) = A(x_1) \cap A(x_2)$.

Q.2(C) Answer any one. (03)

(1) In usual notations prove that $(a * b)' = a' \oplus b'$.

(2) In usual notations prove that $A(x') = A - A(x)$

Q.3(A) Answer any one. (07)

(1) In usual notations obtain Cauchy – Riemann conditions in polar form.

(2) If f(z) = u + i v is an analytic function on domain D then prove that

$$\left(\frac{\partial|f(z)|}{\partial x}\right)^2 + \left(\frac{\partial|f(z)|}{\partial y}\right)^2 = |f'(z)|^2$$

Q.3(B) Answer any one. (04)

(1) Prove that u = log r is a harmonic function and find it's conjugate.

(2) Prove that an analytic function of a constant modulus is also constant in its domain.

Q.3(C) Answer any one. (03)

(1) Prove that $\frac{1}{z}$ is an analytic function but not an entire function.

(2) Show that z^2 is entire function.

Q.4(A) Answer any one. (07)

(1) State and prove Cauchy's integral formula.

(2) Show that $\left|\int_C \frac{z+4}{z^3-1}\, dz\right| \le \frac{6\pi}{7}$ Where C be the arc of the circle $|z| = 2$ from z = 2 to z = 2i.

Q.4(B) Answer any one. (04)

(1) In usual notations prove that $\left|\int_C f(z)dz\right| \le$ ML.

(2) Find: $\int_C \frac{z^2+3}{z^2(z-4)}$ dz where C: |z| = 1.

Q.4(C) Answer any one. (03)

(1) Find $\int_C \frac{z}{(z+i)}$dz; where C: $|z| = 2$.

(2) Find: $\int_C \frac{dz}{(z-3)}$ where C: | z - 2| = 5.

Q.5(A) Answer any one. (07)

(1) State and prove the fundamental theorem of algebra.

(2) State and prove Morera's theorem.

Q.5(B) Answer any one. (04)

(1) Find: $\int_C \frac{5z^2-7z+3}{(z-1)^2}$ dz where C: |z| = 2

(2) In usual notations state and prove Cauchy's inequality.

Q.5 (C) Answer any one. (03)

(1) If C: |z| = 3 is the closed contour and the integral is fallen in positive sense and g(z) $= \int_C \frac{2s^2-s-2}{s-z}$ ds then prove that g(2) = 8πi.

(2) Find: $\int_C \frac{\cosh z}{z^3} dz$ where C: | z | = 1.

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